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A multi-time-step noise reduction method for measuring velocity statistics from particle tracking velocimetry

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Abstract

We present a method to improve the accuracy of velocity measurements for fluid flow or particles immersed in it, based on a multi-time-step approach that allows for cancellation of noise in the velocity measurements. Improved velocity statistics, a critical element in turbulent flow measurements, can be computed from the combination of the velocity moments computed using standard particle tracking velocimetry (PTV) or particle image velocimetry (PIV) techniques for data sets that have been collected over different values of time intervals between images. This method produces Eulerian velocity fields and Lagrangian velocity statistics with much lower noise levels compared to standard PIV or PTV measurements, without the need of filtering and/or windowing. Particle displacement between two frames is computed for multiple different time-step values between frames in a canonical experiment of homogeneous isotropic turbulence. The second order velocity structure function of the flow is computed with the new method and compared to results from traditional measurement techniques in the literature. Increased accuracy is also demonstrated by comparing the dissipation rate of turbulent kinetic energy measured from this function against previously validated measurements.

Keywords: noise reduction, flow velocimetry measurements, particles/tracer dynamics, PTV/PIV, turbulent structure function, turbulent dissipation rate

 Supplementary material for this article is available [online](#)

(Some figures may appear in colour only in the online journal)

1. Introduction

Flow velocity measurements, based on the analysis of the motion of particles imaged with digital cameras, have become the most commonly used metrology technique in contemporary

fluid mechanics research [1]. Particle image velocimetry (PIV) and particle tracking velocimetry (PTV) are two widely used methods that enable the characterization of a flow from the Eulerian (PIV) or Lagrangian (PTV) point of view.

PTV is a versatile measurement technique to probe complex fluid mechanics problems, when the particles are fluid tracers, and particle-fluid interactions, when the particles have

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inertia and do not follow the flow [2–5]. Recently, multi-camera approaches and wider illumination regions have become widely available for particle tracking, enabling measurement of the three components of the velocity in a relatively thin volume (3C–2D) and fully three dimensional measurements of the three components of the velocity at a prismatic volume in the flow (3C–3D). Two frequent realizations of this method in the laboratory are based on taking a pair of images using double exposure cameras, typical of PIV, at a very short time separation followed by a large time interval, or collecting a long sequence of images closely and equally separated in time using high speed cameras. The first technique provides a single vector per particle in a pair of consecutive images, with subsequent velocity measurements in other image pairs being uncorrelated from this. The high speed image sequence, on the contrary, provides the opportunity to track the same particle over multiple images and provides several correlated velocity measurements at different locations but along the same particle trajectory.

Noise reduction in velocity measurements based on the displacement of particles from images of scattered light have been developed and tested extensively. From tracking detection from multiple sequential images [6–9] to improved centroid detection [10], error estimation [11] and camera calibration [12], the advantages of noise-reduction on imaging or particle detection for improved accuracy of velocimetry have been demonstrated. This note takes a completely different approach to noise-reduction. The novel method presented here removes noise not from the individual image or particle center detection, but globally in the calculation of velocity statistics by using the redundant information in measurements taken from image sequences collected at multiple time steps. The presentation uses as an example the computation of Eulerian velocity statistics by spatial binning in order to determine flow features and turbulence characteristics (such as dissipation).

2. Background on particle location measurement noise

Experimental noise in the position of particles in an image is particularly critical for measurements of the velocity or acceleration fields, where differentiation is needed. PTV introduces noise both from the light scattering and imaging physics, and from the tracking/correlation mathematical processing of those positions [1, 13].

In PTV, particle positions are determined, including measurement noise, by detecting the centroid of each region of the camera sensor that records light intensity above a certain threshold in contiguous pixels. While the uncertainty in the detection of the centroid as the true location of the particle center does not represent a significant problem in the computation of particle location statistics, the noise in the particle location becomes a severe limitation when using these measurements to calculate velocity or acceleration (first and second derivatives, respectively). The need to filter the particle positions prior to any differentiation has long been recognized [4, 14–16]. Multiple filter parameters, for example filter length,

have to be chosen or adjusted to minimize the propagation and amplification of noise in the process of taking the derivative of the measurements. How the derived quantities depend on the filtering can be studied to provide confidence in the data, even leading to information on the expected value of some moments of the measurement distribution (the rms value of particle acceleration in [14, 15]). Filtering, however, generally removes information, erasing events (changes in particle trajectories) with time scales shorter than the filter length.

If the particle position is $\mathbf{X}(t)$ and the Lagrangian displacement field is $\Delta\mathbf{X} = \mathbf{X}(t + \Delta t) - \mathbf{X}(t)$, noise \mathbf{b} in the detected position $\tilde{\mathbf{X}}$ biases the statistics computed using a naive definition of the velocity $\Delta\mathbf{X}/\Delta t$. The method described here computes noiseless n th moments and n th order structure functions of the velocity field, assuming white noise: $\langle \mathbf{b}(t)\mathbf{b}(t + \Delta t) \rangle_t = \langle \mathbf{b}^2 \rangle_t \delta(\Delta t)$, $\langle \mathbf{b}\Delta\tilde{\mathbf{X}} \rangle_t = \langle \mathbf{b} \rangle_t \langle \Delta\tilde{\mathbf{X}} \rangle_t = 0$, where $\langle \rangle_t$ is an average over time.

The displacement field being $\Delta\mathbf{X} = \Delta\tilde{\mathbf{X}} + \Delta\mathbf{b}$, its second moment can be expressed as

$$\langle (\Delta\mathbf{X})^2 \rangle_t = \langle \tilde{\mathbf{v}}^2 \rangle_t \Delta t^2 + 2 \langle \mathbf{b}^2 \rangle_t + \langle \tilde{\mathbf{a}}\tilde{\mathbf{v}} \rangle_t \Delta t^3 + o(\Delta t^3). \quad (1)$$

3. Noise-removal algorithm

The binned Lagrangian displacement (subtracted of the time-averaged displacement in each bin, free of white noise as shown above), gives an Eulerian field of fluctuating displacement to compute the velocity structures in time or space.

To remove the noise from the velocity statistics, $\langle \Delta\mathbf{X} \rangle_t$ and $\langle (\Delta\mathbf{X})^2 \rangle_t$ are calculated for multiple experimental sets where images of the particles in the flow are collected at increasing values of Δt . When the evolution of $\langle (\Delta\mathbf{X})^2 \rangle_t$ with Δt is fitted by a polynomial of the form $c_1 \Delta t^2 + c_2$, the leading coefficient is the velocity variance $\langle \tilde{\mathbf{v}}^2 \rangle_t$. The third order correction is negligible because dimensional analysis gives $\langle \tilde{\mathbf{v}}^2 \rangle_t / \langle \tilde{\mathbf{a}}\tilde{\mathbf{v}} \rangle_t \tau_\eta \sim \text{Re}_\lambda$, where $\tau_\eta = \sqrt{\nu/\varepsilon}$ is the dissipative time and Re_λ is the Reynolds number at the Taylor-length scale. In turbulent flows, $\langle \tilde{\mathbf{a}}\tilde{\mathbf{v}} \rangle_t$ is well approximated by the dissipation rate ε . Taking Δt smaller than the dissipative time ensures that the displacement field variance is well approximated. The advantage of this method is that it uses all the measurements taken at different values of Δt without having to choose any particular Δt , optimized according to some metric.

This method can be extended to higher order moments of the displacement field, as well as to recover increment statistics, for example the longitudinal second order structure function of the velocity ($\tilde{S}_2 = \langle [(\tilde{\mathbf{v}}(\mathbf{X} + \mathbf{r}) - \tilde{\mathbf{v}}(\mathbf{X})) \cdot \mathbf{e}_r]^2 \rangle$ (where \cdot is a scalar product and $\langle \rangle$ an ensemble average), with $\mathbf{e}_r = \Delta\mathbf{X}/|\Delta\mathbf{X}|$), by fitting the evolution of $\langle [(\Delta\mathbf{X}(\mathbf{X} + \mathbf{r}) - \Delta\mathbf{X}(\mathbf{X})) \cdot \mathbf{e}_r]^2 \rangle$ with a polynomial $\tilde{S}_2(|\mathbf{r}|)\Delta t^2 + c_2$:

$$\begin{aligned} \langle [(\Delta \mathbf{X}(\mathbf{X} + \mathbf{r}) - \Delta \mathbf{X}(\mathbf{X})) \cdot \mathbf{e}_r]^2 \rangle &= \langle [(\Delta \mathbf{b}(\mathbf{X} + \mathbf{r}) - \Delta \mathbf{b}(\mathbf{X})) \cdot \mathbf{e}_r]^2 \rangle \\ &+ \langle [(\tilde{\mathbf{v}}(\mathbf{X} + \mathbf{r}) - \tilde{\mathbf{v}}(\mathbf{X})) \cdot \mathbf{e}_r]^2 \rangle \Delta t^2 \\ &+ \langle [(\tilde{\mathbf{v}}(\mathbf{X} + \mathbf{r}) - \tilde{\mathbf{v}}(\mathbf{X})) \cdot \mathbf{e}_r][\tilde{\mathbf{a}}(\mathbf{X} + \mathbf{r}) - \tilde{\mathbf{a}}(\mathbf{X})) \cdot \mathbf{e}_r] \rangle \Delta t^3 + o(\Delta t^3). \end{aligned} \quad (2)$$

Note that the structure function computation does not require the conversion of displacements to Eulerian coordinates, but rather to bin the inter-particle distance $|\mathbf{r}|$. This means that measuring structure functions is possible at arbitrarily small separations $|\mathbf{r}|$, without any requirements on the Eulerian spatial binning. This method requires only a statistical convergence in the number of particles N at a certain range of inter-particle distance (a number that goes with N^2). This represents a significant advantage over methods for structure function computation that carry an associated increase in measurement noise at small separations $|\mathbf{r}|$.

The second order moment of the velocity fluctuations and second order structure function are presented here as examples of what the expansion of statistical moments combined with data collected at different Δt can achieve. Higher order moments for the velocity fluctuations and higher order structure functions can be easily computed by this method with reduced noise, although they will contain residual noise from the computation of lower order moments ($o(\Delta t^3)$ terms above).

4. Results

Particle displacements measured in a homogeneous, isotropic turbulence experiment [17, 18] are used to demonstrate the validity and accuracy of the method. Two CMOS cameras with a resolution of 2048×1088 pixels were used in a stereoscopic arrangement. Images were collected in double-frame mode, separated by a time-step Δt from $0.05\tau_\eta$ to $0.2\tau_\eta$. Alternatively, a very fast acquisition/illumination rate using a high speed camera and kHz pulsed lasers allows the collection of a single image sequence and then taking a variable Δt in the analysis by skipping an increasing number of images in the sequence. Measurements were obtained in a volume of $10 \times 10 \times 1 \text{ cm}^3$ using a Nd:YAG laser. For each experiment, approximately 10000 pairs of image sets per time-step (each set providing the 3D position of several hundred particles in the flow) were collected to ensure statistical convergence. More details can be found in the supplementary material (stacks.iop.org/MST/28/107002/mmedia).

The different results for the longitudinal second order structure function of $\Delta \mathbf{X}$ (figure 1(a)) at different time steps, Δt_i , show in their shape a strong dependency on how the noise affects the signal for different values of Δt . The displacement correlation plotted at fixed separations (five different values) are all quadratic in Δt (figure 1(b)), showing this approximation is robust for different levels of measurement noise. The trend $c_1\Delta t^2 + c_2$ from equation (2) is followed at different values of the separation $|\mathbf{r}|$, with the positive values of c_2 being proportional to the variance of the noise (equation (1)). The quadratic coefficient c_1 is the second order function of the velocity with the noise removed. The presence of the inertial range is highlighted by the 2/3 slope in figure 2(a), over approximately one decade, in good agreement with

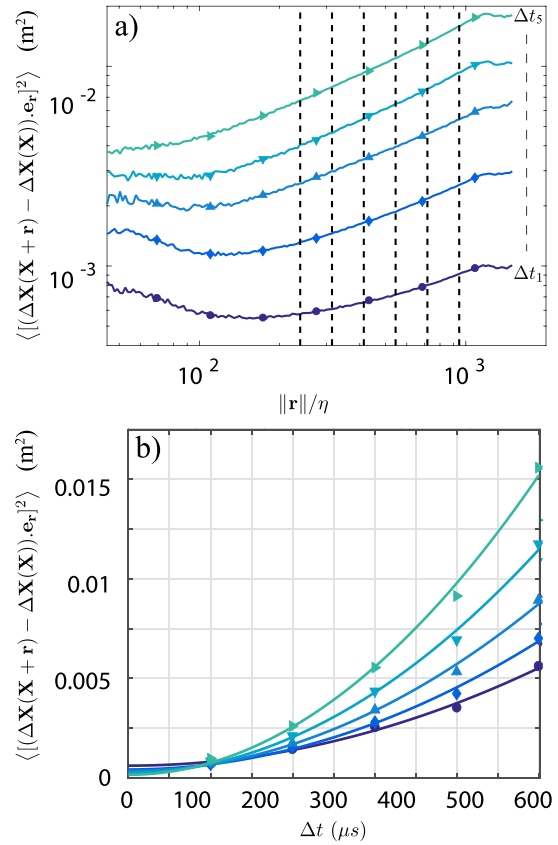


Figure 1. (a) Longitudinal second order structure functions of the raw displacement field $\Delta \mathbf{X}$ against the separation $|\mathbf{r}|$ normalized by the Kolmogorov length scale η for different values of Δt_i equally spaced from 0.05 to $0.2\tau_\eta$ at $\text{Re}_\lambda = 291$. (b) Same quantities but plotted at a given separation $|\mathbf{r}|$ (indicated by the vertical dashed lines on (a); ascending order is for different values of Δt), as a function of the inter-frame time-step value Δt . The lines are fits of the form $c_1\Delta t^2 + c_2$.

Kolmogorov's prediction for the second order structure function in homogeneous isotropic turbulence ($\tilde{S}_2 \sim \varepsilon^{2/3}|\mathbf{r}|^{2/3}$) [19]. Turbulence variables extracted from velocity measurements would be subject to a significant level of uncertainty and inaccuracy (seen in figure 1(a)) if the noise were not removed by the method proposed here.

Figure 2(b) shows the estimation of the dissipation rate of turbulent kinetic energy, $\varepsilon_r = \tilde{S}_2^{3/2}/|\mathbf{r}|$ for three different Reynolds numbers studied in this experimental implementation of this denoising method. The plateaux found in the decade of separation values shown confirm the presence of the inertial range and that their values correspond to the ensemble average of the local dissipation rate. The estimations of ε , as well as u' (spatial average of the fluctuating velocity map), for different Reynolds numbers compare well with those in [18], obtained by 2D3C PIV, confirming the accuracy of the method. In fact, the values of u' and ε are slightly lower than those obtained by PIV. This discrepancy can be explained, qualitatively, based on the physics of the measurements and the effect of the noise on these metrics when it is not eliminated from the displacement measurements. Previous velocity measurements in the same experiment, conducted by traditional PIV [18], corresponded well with the actual velocity

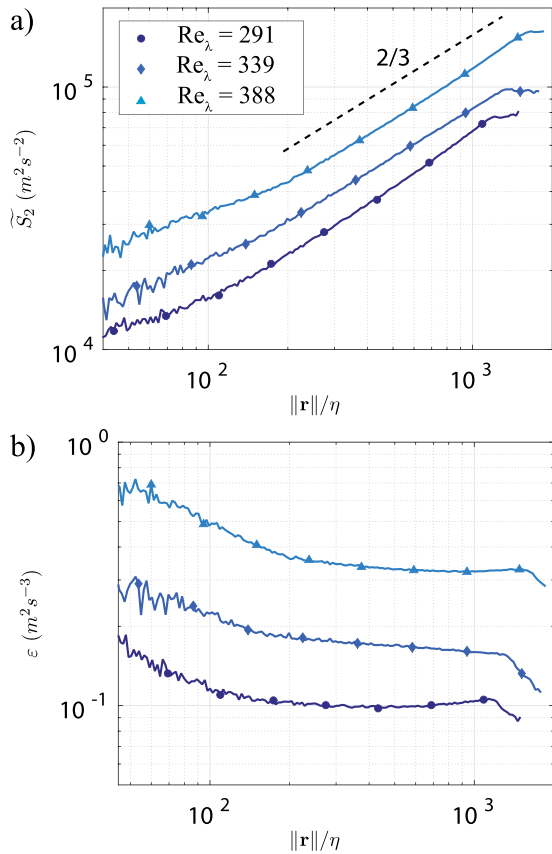


Figure 2. (a) Second order structure functions of the velocity extracted with the proposed method for different Re_λ . The black dashed line corresponds to a power law of exponent $2/3$. (b) Energy dissipation rate estimated as $\varepsilon_r = \tilde{S}_2^{3/2}/|\mathbf{r}|$.

measured with this technique, but with noise variance convoluted onto it. The structure function (and hence ε) measured with traditional techniques was also subject to this erroneous increase in the value due to the creep-in of noise into the computation of this statistical value. Equation (2) shows that the term $\langle [(\Delta \mathbf{b}(\mathbf{X} + \mathbf{r}) - \Delta \mathbf{b}(\mathbf{X})) \cdot \mathbf{e}_r]^2 \rangle$ will increase the value ε due to noise. To determine the importance of this term, it has to be expanded into $4 \langle \mathbf{b}^2 \rangle (1 - C_b(|\mathbf{r}|))$, where $C_b(|\mathbf{r}|)$ is the noise spatial correlation, bounded between $(-1, 1)$. Regardless of the value of C_b , it will increase erroneously the value of the structure function yielding a higher value of ε , and because the value of C_b depends on spatial separation, it will not raise it uniformly for all values of $|\mathbf{r}|$, changing the slope of the structure function with separation, thus making the value of ε noisier.

5. Discussion

The comparison of the flow statistics with a previous 2D3C PIV study [18] allows for the validation of the proposed method. In fact, the measurements show better results, with no need to tune arbitrary filtering parameters to remove noise (the interrogation window size for instance). The only parameters that have to be chosen for the new method proposed here are the different values of Δt that are accessible for a given flow and camera/illumination available, the form of the fit function

and finally the binning in space to compute the Eulerian average and fluctuating velocities (if so desired), and in separation distance to compute the structure function.

The values of Δt are subject to two limitations. They have to be high enough so that particles move more than the measurement error, while keeping the large displacements associated with highest Δt from interfering with the ability of the particle tracking algorithm to identify individual particles [20]. As mentioned in the previous section, a maximum value of $\Delta t \lesssim \tau_\eta$ ensures that the third order correction (section 2) remains small ($u'^2 / \langle \tilde{\mathbf{a}}\tilde{\mathbf{v}} \rangle_t \tau_\eta \sim Re_\lambda$). This was verified in the present experimental set-up, and we found this correction to be negligible compared to the second order term. This was also the case for the structure function provided the separation lies in the inertial range $|\mathbf{r}| \gg \eta$. In such cases, the best agreement between fit functions and the data overall was found when using a quadratic function of Δt . As for the number of time-step values needed, the value of ε when using only the three larger values of Δt was only 5% lower than when using all five datasets. Using only the lowest value and largest values of Δt allowed for a simple calculation of ε that was only 2% higher than with the full experimental set.

The displacement vector field obtained from particle tracking in this multi-time-step method is computed in a Lagrangian frame of reference. To compute the values of $\langle (\Delta \mathbf{X})^2 \rangle_t$ against Δt , the displacement field must be binned into a spatial grid, converting it to an Eulerian frame of reference. Although the number of particles per image, or Eulerian grid cell, is relatively small in these PTV images, the velocity is estimated independently for each particle pair. Thus, the statistical convergence in the method is reached relatively quickly (without the need for a very large number of image pairs). The computation of the structure functions highlights this advantage. As pointed out above (section 3), the structure function could in principle be computed to an arbitrarily small separation between particles. However, great care has to be taken in doing so because (i) difficulty to achieve statistical convergence in finding particles with small separations; (ii) the second and third order terms in equation 2 will become of the same order of magnitude as the separation enters the dissipation range ($|\mathbf{r}| \sim 10\eta$). These explain why an increase of the structure functions at small separations is observed in figure 1(a).

This note presents a new method to remove measurement noise from the calculation of velocity statistics, without filtering. It has been tested to compute the value of the energy dissipation rate through the second order structure function of the velocity in an experiment imaging tracer particles in a turbulent flow via stereoscopic particle tracking. The flow statistics computed are in good agreement with a previous 2D3C PIV study, with improved better accuracy.

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References

- [1] Adrian R J 1991 Particle-imaging techniques for experimental fluid mechanics *Annu. Rev. Fluid Mech.* **23** 261
- [2] Sato Y and Yamamoto K 1987 Lagrangian measurement of fluid-particle motion in an isotropic turbulent field *J. Fluid Mech.* **175** 183–99
- [3] Virant M and Dracos T 1997 3D PTV and its application on Lagrangian motion *Meas. Sci. Technol.* **8** 1539–52
- [4] Voth G A, Porta A L, Crawford A, Alexander J and Bodenschatz E 2002 Measurement of particle accelerations in fully developed turbulence *J. Fluid Mech.* **469** 121
- [5] Tropea C, Yarin A and Foss J 2007 *Springer Handbook of Experimental Fluid Dynamics* (Berlin: Springer) pp 789–99
- [6] Xu H 2008 Tracking Lagrangian trajectories in position–velocity space *Meas. Sci. Technol.* **19** 075105
- [7] Ouellette N T, Xu H and Bodenschatz E 2006 A quantitative study of three-dimensional Lagrangian particle tracking algorithms *Exp. Fluids* **40** 301–13
- [8] Cierpka C, Lütke B and Kähler C J 2013 Higher order multi-frame particle tracking velocimetry *Exp. Fluids* **54** 1533
- [9] Schanz D, Gesemann S and Schröder A 2016 Shake-The-Box: Lagrangian particle tracking at high particle image densities *Exp. Fluids* **57** 1–27
- [10] Ouellette N T 2006 Probing the statistical structure of turbulence with measurements of tracer particle tracks *PhD Thesis* Cornell University
- [11] Scharnowski S and Kähler C J 2016 Estimation and optimization of loss-of-pair uncertainties based on PIV correlation functions *Exp. Fluids* **57** 1–11
- [12] Machicoane N, Lopez-Caballero M, Bourgoïn M, Aliseda A and Volk R 2016 Improvements on particle tracking velocimetry: model-free calibration and noiseless measurement of second order statistics of the velocity field arXiv:1605.03803
- [13] Toschi F and Bodenschatz E 2009 Lagrangian properties of particles in turbulence *Annu. Rev. Fluid Mech.* **41** 375
- [14] Mordant N, Crawford A and Bodenschatz E 2004 Experimental Lagrangian acceleration probability density function measurement *Physica D* **193** 245–51
- [15] Volk R, Mordant N, Verhille G and Pinton J-F 2008 Laser Doppler measurement of inertial particle and bubble accelerations in turbulence *Eur. Phys. Lett.* **81** 34002
- [16] Berg J, Ott S, Mann J and Luthi B 2009 Experimental investigation of Lagrangian structure functions in turbulence *Phys. Rev. E* **80** 026316
- [17] Zimmermann R, Xu H, Gasteuil Y, Bourgoïn M, Ans J R V and Bodenschatz E 2010 The Lagrangian exploration module: an apparatus for the study of statistically homogeneous and isotropic turbulence *Rev. Sci. Instrum.* **81** 055112
- [18] Fiabane L, Zimmermann R, Volk R, Pinton J-F and Bourgoïn M 2012 Clustering of finite-size particles in turbulence *Phys. Rev. E* **86** 035301(R)
- [19] Kolmogorov A N 1941 Equations of turbulent motion in an incompressible fluid *Dokl. Akad. Nauk SSSR* **30** 299–303
- [20] Ouellette N T, Xu H, Bourgoïn M and Bodenschatz E 2006 An experimental study of turbulent relative dispersion models *New J. Phys.* **8** 109