The influence of glottal cross-section shape on theoretical flow models (L)

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Physical and mathematical phonation models commonly rely on a quasi-one-dimensional flow model. The assumption of quasi-one-dimensional flow through a glottis with fixed length is analyzed for different cross-section shapes: Circle, rectangle, ellipse, and circular segment. A simplified flow model is formulated which accounts for kinetic losses, viscosity, and cross-section shape. It is seen that the cross-section shape cannot be neglected since it alters boundary layer development and hence the viscous contribution to the pressure drop across the glottis. The commonly applied quasi-one-dimensional flow model is shown to be inaccurate, indicating the potential benefit of taking into account the cross-section shape. © *2013 Acoustical Society of America*. [http://dx.doi.org/10.1121/1.4813397]

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I. INTRODUCTION

Theoretical flow models are widespread in physical phonation modeling since it allows one to express key phonation parameters, such as the phonation pressure threshold or oscillation frequency, as functions of a limited number of physiologically meaningful parameters.[1,2]

Applied theoretical flow models rely on a non-dimensional analysis of the governing Navier-Stokes equations while accounting for typical values of physiological, geometrical, and flow characteristics for normal phonation by a male adult.[3] Resulting non-dimensional numbers (Mach number, Reynolds number, Strouhal number, and mean aspect ratio) allow one to treat the glottal flow as incompressible, laminar, inviscid, quasi-steady, and one-dimensional. Nevertheless, experimental validation of theoretical flow models on rigid[4–6] and deformable[7,8] glottal replicas tends to show that viscous effects due to boundary layer development cannot be neglected. Therefore, the one-dimensional model approach is extended to a quasi-one-dimensional model accounting for viscous flow effects based on the assumption of a quasi-one-dimensional flow in a rectangular glottal area with fixed length.[5,7] Even so, viscous boundary layer development, and hence the viscous contribution to the pressure drop across the glottis, is expected to depend on the shape of the glottal area.[9] Visualization of the auto-oscillation of deformable glottal replicas supports the deflection from a rectangular area during experimental validation.[10] Consequently, the assumption of a rectangular glottal area can be questioned for normal as well as pathological geometrical conditions.

The current paper analyzes the influence of the glottal area shape on the pressure distribution within the glottis. Analytical solutions are favored in order to allow integration in physical or mathematical phonation models.

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II. GLOTTAL CROSS-SECTION SHAPE

The glottal geometry is fully defined by the cross-section shape and the area variation along the main flow direction $A(x)$. In order to allow the use of the cross-section shapes in quasi-analytical models only shapes for which the main geometrical parameters can be expressed analytically are assessed:[9] Rectangle (re), circle (cl), ellipse (el), and circular segment (cs). The different cross-section shapes, depicted in Fig. 1, are, although a severe approximation, relevant to describe the glottal cross-section shape in the case of normal geometrical conditions during respiration or phonation.[3] The cross-section is positioned in the (y, z) plane where y denotes the anterior–posterior and z the lateral direction.

III. GLOTTAL FLOW MODEL

Based on a non-dimensional analysis of the governing Navier-Stokes equations and typical values of geometrical and flow characteristics for normal phonation by a male adult, the flow is assumed to be laminar [$Re \sim O(10^3)$], steady

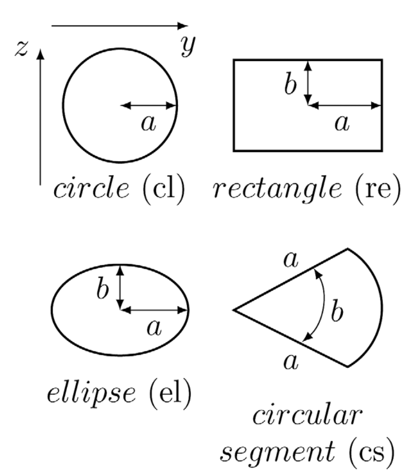


FIG. 1. Different cross-section shapes in the (y, z) plane.

TABLE I. Analytical solutions Q as a function of the pressure gradient dP/dx.

Shape	Volume flow rate $Q\left(\frac{dP}{dx}\right)$
Circle	$\frac{\pi a^4}{8\mu}\left(-\frac{dP}{dx}\right)$
Ellipse	$\frac{\pi}{4\mu}\left(-\frac{dP}{dx}\right)\frac{a^3b^3}{a^2+b^2}$
Rectangle[a]	$\frac{4a^3}{3\mu}\left(-\frac{dP}{dx}\right)\left[b-\frac{192a}{\pi^5}\sum_{n=1,3,\ldots}^{\infty}\frac{\tanh(n\pi b/2a)}{n^5}\right]$
Circular segment[a]	$\frac{a^4}{4\mu}\left(-\frac{dP}{dx}\right)\left[\frac{\tan b-b}{4}-\frac{32b^4}{\pi^5}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^2(n+2b/\pi)^2(n-2b/\pi)}\right]$
BP[b]	$\frac{l_g w^3}{12\mu}\left(-\frac{dP}{dx}\right)$

[a]Infinite sum is limited to $n \leq 60$.
[b]Quasi-one-dimensional approach: Glottal width w and fixed glottal length l_g.

(Strouhal number $Sr \ll 1$), and incompressible (squared Mach number $Ma^2 \ll 1$).[5] For air with density $\rho = 1.2\,\mathrm{kg/m^3}$ and kinematic viscosity $\nu = 1.5\times10^{-5}\,\mathrm{m^2/s}$, the streamwise momentum equation of the governing Navier-Stokes equations is further simplified using volume flow rate conservation, i.e., $dQ/dx = 0$, as

$$-\frac{Q^2}{A^3}\frac{dA}{dx} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{1}$$

with driving pressure gradient dP/dx, local velocity $u(x, y, z)$, cross-section area $A(x)$, and volume flow rate Q. The flow model expressed in Eq. (1) accounts for viscosity (second right-hand term) as well as kinetic losses (source term at the left-hand side) and depends therefore on the area as well as the shape of the cross-section. Depending on driving pressure and geometry, in particular minimum area $A_{\min}$, viscous boundary layer development affects the flow development so that a three-dimensional aspect is added to the model. Classical simplified flow models make a quasi-one-dimensional flow assumption by neglecting the anterior–posterior dimension so that the first term within parentheses of Eq. (1) is dropped. This results in the common quasi-one-dimensional flow model (BP) assuming a rectangular cross-section with fixed glottal length l_g and variable glottal width w.[5,7]

In addition, for $dA/dx = 0$, Eq. (1) reduces to the Poisson equation,

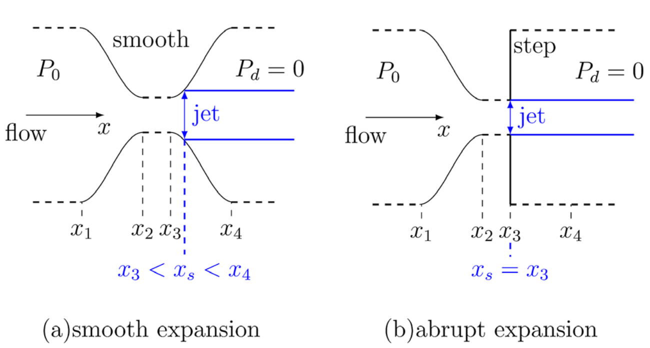


FIG. 2. (Color online) Flow within a converging–diverging geometry with upstream area A_0 and minimum area $A_{\min}$ for (a) a smooth and (b) an abrupt expansion.

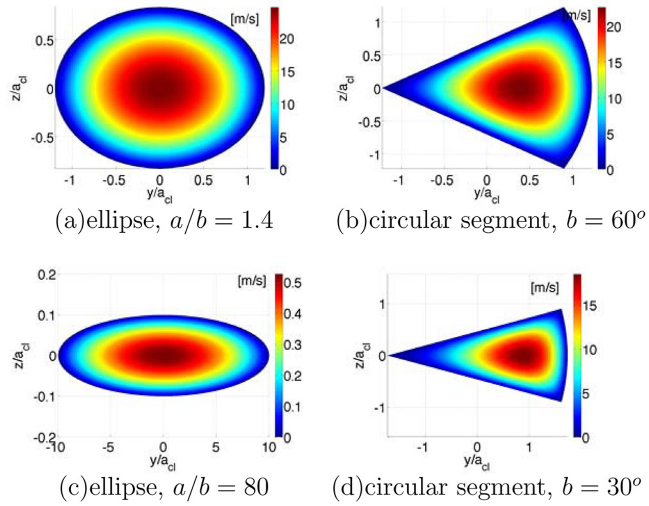


FIG. 3. (Color online) Velocity distributions $u(y/a_{cl}, z/a_{cl})$ for $A = 79\,\mathrm{mm^2}$, $dP/dx = 75$ Pa/m, and different parameter sets $\{a, b\}$.

$$\nu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) = -\frac{1}{\rho}\frac{dP}{dx}, \tag{2}$$

describing purely viscous parallel flow through a uniform channel with arbitrary but constant cross-section shape. For uniform geometries and applying the no slip boundary condition, $u = 0$, on the channel walls, Eq. (2) can be rewritten as a classical Dirichlet problem which can be solved analytically for simple geometries using, e.g., separation of variables or conformal mapping. Therefore exact solutions can be obtained for: Local velocity $u(y, z)$, local pressure $P(x)$, wall shear stress $\tau(x)$, and derived quantities such as volume flow rate Q and bulk Reynolds number $Re = QD/\nu A$ with hydraulic diameter D. Analytical solutions for the volume flow rate Q as a function of the driving pressure gradient dP/dx and geometrical parameters a and b are given in Table I.

Neglecting viscosity, $\nu = 0$ as for an ideal inviscid fluid, reduces Eq. (1) to Euler's equation,[5,9] labeled Bernoulli flow (B),

$$u_b\frac{du_b}{dx} = -\frac{1}{\rho}\frac{dP}{dx}, \tag{3}$$

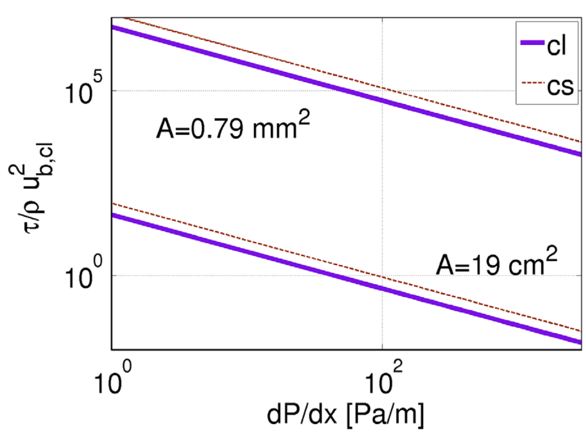
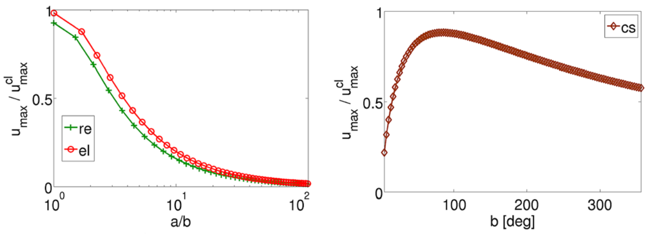


FIG. 4. (Color online) Normalized wall shear stress τ as a function of dP/dx, area A and cross-section shape (circular segment with $b = 60°$).



(a)re, el (b)cs

FIG. 5. (Color online) Influence of cross-section shape parameters on normalized maximum velocity, u_{max}/u_{max}^{cl}, for fixed area and pressure gradient: (a) Rectangle and ellipse and (b) circular segment.

with bulk velocity $u_b(x)$ so that volume flow rate $Q = A(x)u_b(x)$. Equation (3) describes one-dimensional flow accounting for kinetic losses due to a streamwise variation of the glottal cross-section area as depicted in Fig. 2. The main effects of a converging–diverging area variation on the flow are important flow acceleration in the constricted portion and the occurrence of jet formation associated with flow separation along the divergent portion. For an abrupt expansion characterized by a sharp trailing edge, the streamwise position of flow separation x_s is fixed at the constriction end, so that $x_s = x_3$ as in Fig. 2(b). For a smooth expansion, the flow separation position depends on the channel geometry as well as on the imposed driving pressure gradient dP/dx, so that $x_3 < x_s < x_4$ as in Fig. 2(a). The simplest way to model the separation position $x = x_s$ is to assume that at the separation position $x = x_s$ the glottal area yields $A(x_s) = c_s \times A_{min}$ where the constant c_s is set to $c_s = 1.2$ in accordance with the literature.[5,7] The pressure downstream from the flow separation position is assumed to be zero so that $P_d = 0$ holds for $x \geq x_s$ and the model outcome remains constant for $x \geq x_s$. Consequently, imposing the upstream pressure P_0 allows the total driving pressure difference to be imposed.

TABLE II. Geometrical condition in addition to a fixed area. Note that circle and square are special cases of the ellipse and rectangle corresponding to $a/b = 1$.

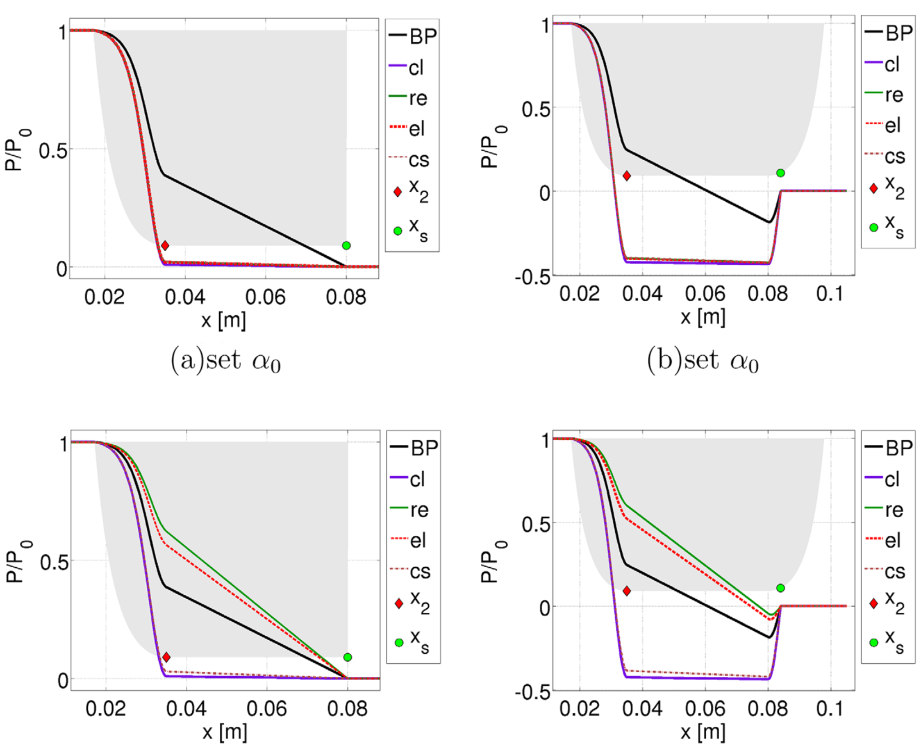
	Ellipse	Rectangle	Circular segment
α_0	$a/b = 1.4$	$a/b = 1.3$	$b = 60°$
α	$a/b = 25$	$a/b = 32$	$b = 30°$

IV. RESULTS

A. Uniform geometry: Cross-section shape

Figure 3 illustrates the modeled influence of the cross-section shape and parameter values $\{a, b\}$ on the velocity distribution $u(y/a_{cl}, z/a_{cl})$ for a fixed area A and dP/dx for a uniform channel following Eq. (2). The normalized mean wall shear stress, illustrated in Fig. 4, depends on the cross-section shape and increases as dP/dx decreases and A decreases.

Figure 5 illustrates the influence of varying the geometrical shape parameters $\{a, b\}$ on the simulated maximum velocity u_{max} for a fixed cross-section area. For a rectangular and elliptic cross-section shape, it is seen that the effect of viscosity increases with the ratio a/b, where $a/b = 1$ corresponds to a square and to a circle respectively. Increasing $a/b \gtrsim 40$ does not influence the effect of viscosity. In the case of a circular segment, increasing the angle b of the segment decreases the influence of viscosity at first until $b \simeq 85°$. Further increasing the angle enforces the influence of viscosity, so that the ratio u_{max}/u_{max}^{cl} decreases. This general tendency reflects the variation of the hydraulic diameter as function of angle b. Two geometrical parameter sets α_0 and α, summarized in Table II, are selected from Fig. 5 for which the influence of viscosity or boundary layer development on the flow is limited (α_0) and pronounced (α), respectively.



(a)set α_0 (b)set α_0 (c)set α (d)set α

FIG. 6. (Color online) Normalized pressure $P(x)/P_0$ for $A_{min}/A_0 = 0.1$ and $P_0 = 1000$ Pa for ideal fluid with quasi-one-dimensional viscous correction (BP) and ideal flow with viscous term function of the cross-section shape for parameter sets α_0 and α, given in Table II, for an abrupt [(a) and (c)] and a smooth [(b) and (d)] expansion. The geometry is depicted in gray shade.

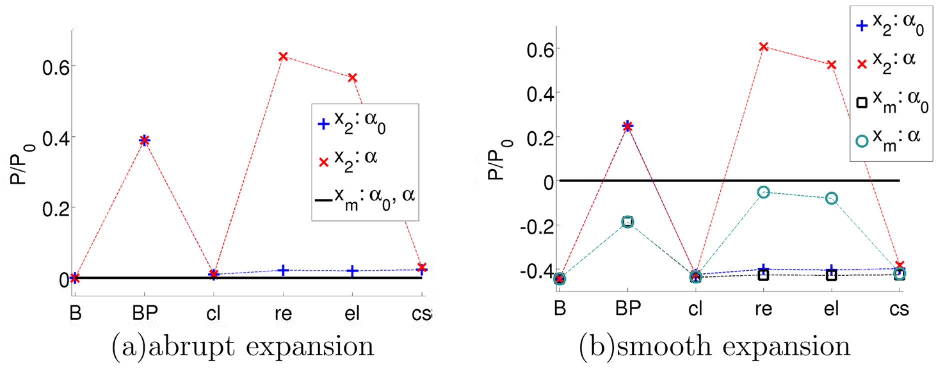


FIG. 7. (Color online) Normalized pressure P/P_0 shown in Fig. 6 at the onset of the minimum area x_2 and at the position of minimum pressure within the constriction x_m for (a) an abrupt and (b) a smooth expansion for an ideal fluid (B), for a quasi-one-dimensional model (BP) and for different parameter sets, α_0 and α given in Table II, and different cross-section shapes: Circle (cl), rectangle (re), ellipse (el), and circular segment (cs).

B. Glottal geometry: Varying streamwise area

Figure 6 illustrates the normalized pressure distribution P/P_0 within a convergent–divergent geometry derived from Eq. (1) for the geometrical conditions summarized in Table II. As a reference, the quasi-one-dimensional flow model outcome (BP) is shown as well. For the parameter set labeled α_0 [Figs. 6(a) and 6(b)] the viscous contribution to the pressure drop is low regardless of the cross-section shape. Therefore, the pressure drop within the uniform portion of the constriction is almost constant as expected for an ideal fluid (B) described by Eq. (3). The quasi-one-dimensional viscous contribution (the line BP) overestimates the pressure loss within the constricted portion by 20% or more. For the parameter set labeled α [Figs. 6(c) and 6(d)] the pressure drop varies again from an almost constant value expected in case of an ideal fluid (circular segment) to well above ($\geq$10%) the quasi-one-dimensional viscous contribution (the line BP), as, e.g., observed for a rectangular or elliptic cross-section.

Figure 7 quantifies the normalized pressure at $x = x_2$, corresponding to the onset of the minimum area, and at $x = x_m$ corresponding to the position of minimum pressure within the constriction. In the case of an abrupt expansion the minimum pressure equals zero regardless of the cross-section shape, whereas variations in the cross-section shape increases the pressure at $x = x_2$ by up to $\leq$60%. In the case of a smooth expansion the impact of the cross-section shape is more pronounced. At $x = x_m$, the minimum pressure P/P_0 is negative and varying the cross-section shape induces a variation by as much as $\leq$40%. At the onset of the constriction $x = x_2$ the influence is even larger since the pressure variation increases to 100%. As for an abrupt expansion, the quasi-one-dimensional model accounting for viscosity (BP) results in a significant underestimation or overestimation of the pressure at $x = x_2$ ($\geq$15%) as well as at $x = x_m$ ($\leq$25%) depending on the cross-section shape.

V. CONCLUSION

Classical phonation models apply a quasi-one-dimensional flow model accounting for kinetic losses and fluid viscosity, where the viscous flow effects are derived assuming quasi-one-dimensional flow through a glottis with fixed length. In the current paper, it is proposed to improve the flow model by no longer making the assumption of quasi-one-dimensional flow. Instead the cross-section shape is taken into account resulting in analytical flow solutions for which the viscous part depends on the cross-section shape. The pressure distribution within the glottal constriction is seen to vary from 20% up to 100% compared to the quasi-one-dimensional flow solution. Since this is of the same order of magnitude as well-studied flow events such as the position of flow separation, the current study suggests that applying the proposed analytical flow model improves the performance of simplified physical or mathematical models of phonation. Experimental studies need to be performed in order to validate the current findings for steady and unsteady flow.

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